

# A Structured Catalogue of Musical Scales

*Music Theory Reference Series*  
*Volume 1*

Heptatonics: A Structured Catalogue of Heptatonic Scales by Nicholas Stylianou

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# Preface

Researching and writing this book has been a tremendously challenging project. Against a backdrop of broad division between the arts and the sciences one encounters a labyrinth of dichotomies in music between practice and theory, popular and art music, tradition and modernity, historical and theoretical musicology, western and non-western ethnomusicology, and others. Many of these dichotomies are deeply embedded and institutionalised within the musical and academic establishment, and while this has undoubtedly enabled extensive progress within specialised fields, it has also made the interactions between these fields increasingly complex. Despite recognition of the problematic nature of these dichotomies, reconciling them remains a hugely difficult task, but one that is nonetheless worthwhile in its aspirations to provide important bridges into and across disciplines.

Conventional western music notation provides a rich arena for such bridges. It has persisted as a means of representing music, not only within the historical and cultural boundaries of the Western tonal tradition in which it evolved, but also well beyond those boundaries. Notation also spans musical practice and theory, and raises interesting questions about their continually evolving interaction. This book explores the structural possibilities and limitations of conventional notation, focusing specifically on seven-note collections, in order to provide a scale-based perspective on the notational choices and interpretations that the modern musician is often faced with.

The overall aim is to reconcile dichotomies by providing a resource that is accessible to professional musicians and amateur enthusiasts of both popular and art music, that conveys theoretical aspects of music in a way that is of practical utility, and does so across historical and sometimes cultural boundaries. This is not to say that the book aspires to one universal ideology that homogenises diversity; rather, it simply recognises that much of this diversity does not exist in isolation, and that while specialised study is immensely important there is also value in exploring the interactions and analogies between different forms of musical thinking.

It is very much hoped that the material presented in this book provides

a useful perspective for a wide range of musicians and musicologists of all backgrounds, and that the results of this perseverance in striving to confront and reconcile some of the dichotomies of music will somehow be helpful for others who are also striving to bridge these gaps.

Overall, the book and its three introductory sections may be approached from different perspectives. Practically oriented readers may choose to dive into the main body of the catalogue, referring to the "Practical Guide" for supporting explanations, and perhaps broadening their interest into the "Musicological Context" and "Theoretical Background" sections in the longer term. Academically oriented readers may prefer to approach the "Musicological Context" first, while theoretically oriented readers may wish to start with the "Theoretical Background", and in each case subsequently broaden their interests into the more practical aspects. Any reader should feel free to use the book as a resource in the way that they feel most comfortable with.

The book's official web-site may be found at:

<http://www.reachcomputing.co.uk/publications/heptatonics.html>

where additional resources may be made available in future.

## Acknowledgements

I would like to thank all the staff and supporters of the British Library for providing a unique environment in which independent scholars can work towards research contributions that would otherwise be impossible to achieve. I'm also grateful to the Institute of Musical Research at the University of London, whose public seminars during the last twelve years have provided a vital link between academia and the wider music research community. My heartfelt thanks also to my family and friends who, although somewhat baffled at times, have nonetheless offered their support and encouragement, without which I could not have sustained the determination and optimism necessary to complete this endeavour.

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# Introduction

## Musicological Context and Motivation

Now well into the twenty-first century, conventional Western music notation finds itself in an interesting and somewhat contradictory position. While musical developments since the mid-nineteenth century have stretched conventions beyond their limits, prompting calls for radical notational reform, these calls have also largely been resisted and, in many ways, conventional Western notation has consolidated its dominant position. Consequently, modern musicians must be familiar with conventional notation while also able to work around its inherent historical biases and features that have become “superfluous...notational artefacts” (Parncutt, 1999, p. 154).

The most prominent of such features is enharmonic distinction – that is, distinguishing between alternative spellings for the same note, such as  $G\sharp$  (G-sharp) or  $A\flat$  (A-flat) for the note between G and A. Such distinctions are determined by context<sup>1</sup>, and while in the Western tradition this context has predominantly been diatonic, the role of diatonicism has changed over the centuries.

## The Rise and Fall of Enharmonic Distinction

In the medieval system of *musica recta* diatonicism was implicit in the form of the hexachord, a sequence of six notes represented by the syllables *ut-re-mi-fa-sol-la* forming consecutive steps *tone-tone-semitone-tone-tone*. From its natural position with *ut* on C for C-D-E-F-G-A, the hexachord could be relocated using the predecessors of the modern accidentals: the ‘soft’ *rotundum* ( $\flat$ ) representing the syllable *fa* and the ‘hard’ *quadratum* ( $\natural$  or  $\sharp$ ) representing the syllable *mi*. In this way, a ‘soft B’ ( $B\flat$ ) as *fa* would place *ut* on F for F-G-A- $B\flat$ -C-D, while a ‘hard B’ ( $B\sharp$ ) as *mi* would place *ut* on G for G-A- $B\sharp$ -C-D-E, as shown in Figure 1(a).

---

<sup>1</sup>A conventional example is raising the G of E-minor (E-G-B) to  $G\sharp$  in E-major (E- $G\sharp$ -B), or lowering the A of F-major (F-A-C) to  $A\flat$  in F-minor (F- $A\flat$ -C).

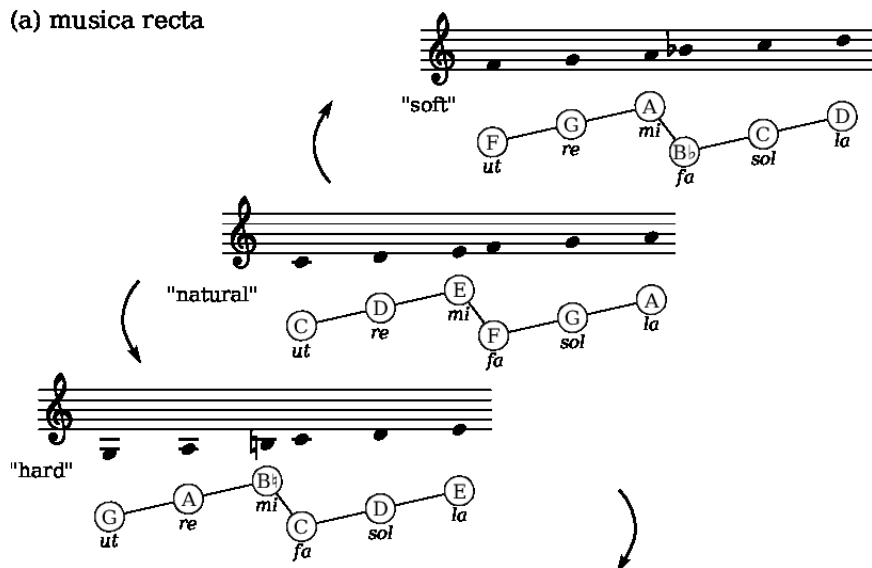
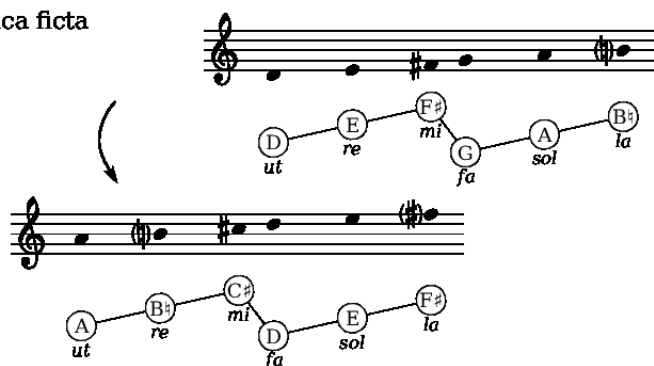
(a) *musica recta*(b) *musica ficta*

Figure 1: The diatonic hexachord in (a) the "natural", "soft" and "hard" positions of *musica recta*, extended by (b) *musica ficta* with examples of "hard F" (F $\sharp$ ) and "hard C" (C $\sharp$ ).

In order to cope with evolving harmonic practice, the system of *musica ficta* extended this to allow the hexachord to be placed in additional positions. In doing so, some inflections of notes were implied by the diatonic structure of the hexachord and did not need to be explicitly notated. For example, a hard C (C $\sharp$ ) as *mi* would place *ut* on A for A-B( $\sharp$ )-C $\sharp$ -D-E-F( $\sharp$ ), in which only the hard C as *mi* need be explicitly notated, the hard B as *re* and hard F as *la* being implied by the structure of the hexachord as whole tones above A as *ut* and E as *sol*. This is shown in Figure 1(b).

From the late fifteenth century onward, a range of musical developments – an increase in the number of polyphonic voices, a shift in emphasis from vocal to keyboard-based music, the revival of modal theory, and the distribution of printed music – prompted significant changes in notational usage (M. Bent and Silbiger, 2001, p. 445). The implicit structure of the hexachord was no longer adequate to describe musical practice and, by the late sixteenth century, the symbols ‘flat’ (b) and ‘sharp’ (#) began to take on their modern meaning as accidentals for lowering and raising pitch by a semitone, with the distinct ‘natural’ (♮) symbol for restoring pitch (I. Bent, 2001, p. 163). With the development of the system of major and minor keys of common-practice tonality through the seventeenth and eighteenth centuries, the notation of accidentals gradually became increasingly explicit.

During the seventeenth century, the double-flat (♭♭) and double-sharp (×) accidentals were introduced. This expanded the scope of enharmonic distinction such that, for example, the note D could also be spelled as *E♭♭* or *C×*<sup>2</sup>, and by the early nineteenth century, conventions of orthography – the correct spelling of notes – became established to the extent that theoretical considerations took precedence over practical convenience (I. Bent, 2001, p. 164). This theoretical basis for enharmonic distinction remained essentially diatonic, with chromatic notes – those outside the diatonic scale – treated firstly as modulation or change of strictly diatonic key (as in the early eighteenth-century theory of Rameau), and subsequently as alteration or extension of diatonic key (Barsky, 1996, pp. 27–29).

However, by the late nineteenth century, there was a growing awareness that some chromatic practice could not be explained in terms of altered diatonicism. Enharmonic distinctions became increasingly ambiguous and their associated theoretical basis of orthography declined in significance. This gave way to enharmonic equivalence rather than distinction and, crucially, the emergence of chromaticism as an independent concept in its own right, breaking free from diatonicism<sup>3</sup>.

The conceptual independence of chromaticism had several consequences: it prompted calls for radical notational reform, stimulated an interest in the expansion and classification of musical resources, and provoked a dichotomy between the concepts of tonality and atonality.

## Notational Reform

The calls for radical notational reform arising from the conceptual independence of chromaticism from diatonicism challenged the “assumption of

<sup>2</sup>See also Theoretical Background: Line of Fifths, p. 35.

<sup>3</sup>This awareness of the conceptual independence of chromaticism has been termed “pitch-class consciousness” (Lansky, 1975 in Bernard, 1997, p. 13).

## Practical Guide

### Overview

This catalogue contains 462 scales; these are all the possible distinct seven-note scale structures that can be formed from an octave of twelve semitones<sup>18</sup>. They are grouped into 66 families of 7 scales each<sup>19</sup>. The families are numbered 1 to 66, and the scales within each family are numbered 1 to 7 – therefore each scale is uniquely identifiable by combining its family number and scale number. For example, §1.3 is family 1 (the ‘Diatonic’ family) scale number 3 (the ‘Ionian’ or major scale).

The 66 families are grouped into 13 parts<sup>20</sup>. These are numbered I to XIII, and broadly form a spectrum from diatonic (part I) to chromatic (part XIII). For an overview of how the families and scales are spread across the 13 parts of the catalogue, see Width Distribution, p. 26.

The remainder of this practical guide describes the properties presented for each scale and family, explains the strategy for naming scales and families, and provides perspectives on the overall organisation of the catalogue in the form of scale/family maps and distributions.

## Scale Properties

### Scale Step Sequence

Each scale has a scale-step sequence. This is unique to each scale and consists of exactly 7 steps, each between 1 and 6 semitones in size and always totalling 12 semitones. For example, §4.1, the ‘Aeolian $\sharp$ 7 (*Harmonic Minor*)’ scale, has a scale-step sequence of ‘2122131’.

In order to locate a scale by its scale-step sequence, a lookup table is provided (see Appendix C: Scale-Step Lookup).

### Scale-Degree Structure



Each scale’s sequence has a corresponding scale-degree structure. This indicates how the sequence of scale-steps corresponds to the positions of the scale’s eight degrees. The 1st and 8th degrees are fixed at the boundaries of an octave of 12 semitones, and the 2nd through to 7th degrees are movable within this range. For example, the scale-step sequence of §4.1 corresponds to a scale-degree structure of ‘1·23·4·56·78’.

<sup>18</sup>Based on Combinatorial Analysis, see Theoretical Background, p. 28.

<sup>19</sup>Based on Rotational Equivalence, see Theoretical Background, p. 30.

<sup>20</sup>Based on their width on the Line of Fifths, see Theoretical Background, p. 35.

## Guitar Pattern

Each scale has a guitar pattern. This is the scale-degree structure split over 3 lines to show the pattern formed by the scale on the guitar fretboard. The patterns are for the top 3 strings of a guitar in standard tuning<sup>21</sup>, and are given for a right-handed guitarist (i.e., left hand on the fretboard) and for a left-handed guitarist (i.e., right hand on the fretboard) indicated by the symbols  and  respectively.

## Example Spelling

Each scale has an example spelling. This represents an instance of the scale in Western musical stave notation. The starting note is always one of the seven natural notes, and has been chosen so that the accidentals on the scale's notes correspond to those in the scale's name (see Scale Names, p. 18). For example, §4.1's starting note of A gives a #7 (*G#*) corresponding to the scale's name 'Aeolian#7'.

For scales with a compound (double or triple) name, the starting note has been chosen to accommodate the accidentals in the most balanced way, minimising bias towards any of the names. For example:

- §2.7's starting note of G gives a b3 (*Bb*) and #7 (*F#*) corresponding to the scale's double name 'Ionianb3/Dorian#7'.
- §7.4's starting note of D gives a b2 (*Eb*) and #7 (*C#*) corresponding to the scale's triple name 'Ionianb3b2/Dorianb2#7/Phrygian#6#7'.

Of course in all cases the reader is free to transpose the example scale to any starting note.

## Symmetry

Each scale has a symmetry relation indicating the scale obtained when the scale-step sequence is reversed. For example, §4.1's scale-step sequence '2122131', when reversed, gives '1312212', the scale-step sequence of §3.7.

Most of the scales in the catalogue are paired in this way (452 scales as  $2 \times 226$  symmetric pairs). The remaining 10 scales of the catalogue are 'self-symmetric' – that is, their scale-step sequence remains the same when reversed – and these are listed in Table 1.

## Similarity (Parents/Siblings/Children)

Each scale has similarity relations indicating the other scales that can be obtained by making a single alteration to the scale-degree structure (i.e., moving just one scale degree). Based on the catalogue's organisation into 13 parts (see Overview, p. 14), these relations are distinguished as parents,

<sup>21</sup>From the top string downwards: E, B and G.

## Theoretical Background

This section provides theoretical detail on some of the principal concepts that underlie this catalogue. In particular:

- Combinatorial Analysis (p. 28) and equivalence relations determine the number of scales and families and the relationships between them.
- The Line of Fifths (p. 35) determines the grouping of the catalogue's families and scales into 13 parts; these provide the organisational basis of the catalogue, forming a hierarchy of parent/sibling/child relations across an overall spectrum from diatonic to chromatic.
- Synthetic Scales (p. 38) are associated with the catalogue's focus on heptatonic scales and how they relate to the concept of alteration in conventional music notation with its diatonic basis.

## Combinatorial Analysis

Combinatorial analysis identifies, for a given set of items, how many different combinations of those items are possible. In this section, the relevant fundamentals are outlined, then applied to the set of twelve pitch-classes before focusing specifically on the seven-note pitch-class combinations that form the basis of this catalogue.

### Fundamentals

For a set of  $n$  items there are a total of  $2^n$  possible ways of combining them. Each combination consists of between 0 and  $n$  items, and it is of interest to identify how many distinct combinations consist of exactly  $r$  items.

Informally, this represents a process in which we start selecting from the set of  $n$  items, one item at a time: the first selection will be any one of the  $n$  possible items, the next selection will be any one of the remaining  $n - 1$  items, and so on, until we have selected  $r$  items, leaving  $n - r$  items unselected. At this point, there are several different sequences by which we could have ended up with the same set of  $r$  items: the first selection could have been any one of those  $r$  items, the second selection could have been any one of the remaining  $r - 1$  items, and so on.

Expressing this mathematically, the number of possible combinations consisting of exactly  $r$  out of the  $n$  items is represented by  ${}^nC_r$  and is calculated<sup>32</sup> as:

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

The elements on the right-hand side of this equation represent three aspects of the process outlined above:

<sup>32</sup>The factorial operator (!) indicates a series of multiplications – for example 3! (“three factorial”) is  $3 \times 2 \times 1 = 6$ , 4! is  $4 \times 3 \times 2 \times 1 = 24$ , etc.

# Chapter 1

## The Diatonic family

**Line-of-fifths pattern** ●●●●●●●●

**Classification** This family corresponds to:

- Forte pitch-class set 7-35 with interval vector 254361.
- Barbour class 1 with scale-step vector 25.

**Symmetry** This family is self-symmetric.

**Parents** This family does not extend any families.

**Siblings** This family is siblings with itself.

**Children** This family is extended by:

- the ‘Ascending Melodic Minor’ family (Ch.2)
- the ‘Harmonic Major’ family (Ch.3)
- the ‘Harmonic Minor’ family (Ch.4)
- the ‘Major  $\flat 2$ ’ family (Ch.5)
- the ‘Neapolitan Minor’ family (Ch.6)
- the ‘Major  $\flat 5$ ’ family (Ch.9)
- the ‘Neapolitan Minor  $\flat 5$ ’ family (Ch.10)

## 1.1 The Aeolian (*Natural Minor*) scale

The scale-step sequence is 2122122 semitones.

The scale-degree structure is 1·23·4·56·7·8

The guitar pattern is:



An example spelling of this scale is:



**Symmetry** This scale is paired with the ‘Mixolydian’ scale (§1.7).

**Parents** This scale does not extend any scales.

**Siblings** This scale is altered by:

- sharpening the 6<sup>th</sup> for the ‘Dorian’ scale (§1.4)
- flattening the 2<sup>nd</sup> for the ‘Phrygian’ scale (§1.5)

**Children** This scale is extended by:

- sharpening the 3<sup>rd</sup> for the ‘Mixolydian♭6/Aeolian♯3’ scale (§2.4)
- flattening the 5<sup>th</sup> for the ‘Aeolian♭5/Locrian♯2’ scale (§2.5)
- sharpening the 7<sup>th</sup> for the ‘Aeolian♯7’ scale (§4.1)
- flattening the 4<sup>th</sup> for the ‘Aeolian♭4’ scale (§5.1)
- sharpening the 4<sup>th</sup> for the ‘Aeolian♯4’ scale (§6.1)
- flattening the 7<sup>th</sup> for the ‘Aeolian♭7’ scale (§9.1)

**Ethnomusicology** This scale approximates the scale structure of South-Indian *mela* 20 ‘Natabhairavi’, North-Indian *thāt* A5 ‘Āsāvī’, Arab *maqāmāt* ‘Nahāwand (Kurdī)’, ‘Būsālīk’, ‘Ushāq Masrī’ and ‘Farahfāzā’, Turkish *makamlar* ‘Uşşak’, ‘Bayatī’ and ‘Nihavent’ and Greek *dromoi* ‘Nisiotiko Minore’, ‘Melodiko Minore (desc.)’ and ‘Houseini (desc.)’.



## Appendix A

### Forte Classification

The 38 heptatonic pitch-class set classes listed in Table A.2 are those given by Forte (Forte, 1973, pp. 179–181). They correspond to all 66 families across all 13 parts of the catalogue, as shown in Table A.1 and Figure A.1, distinguished by the first digit of Forte’s interval-class vector (see Additional Notes below).

Table A.1: The distribution of the catalogue’s 66 families by width, distinguished by the first digit of Forte’s interval-class vector.

Vector	Width													Total
	7	9	10	11	12	14	16	17	19	21	24	26	31	
2	1	1		1										3
3			2	2	6	6	4							20
4				1	2	2	4	6	9	6				30
5								2		2	4	4		12
6													1	1
Total	1	1	2	4	8	8	8	8	9	8	4	4	1	66

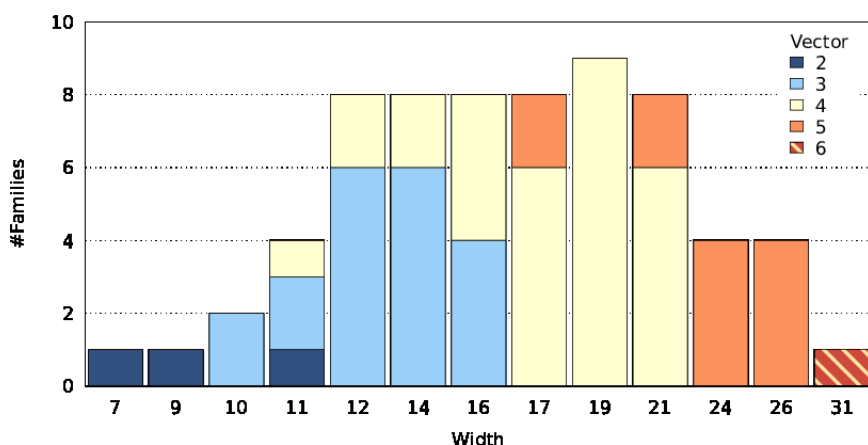


Figure A.1: The distribution of the catalogue’s 66 families by width, distinguished by the first digit of Forte’s interval-class vector.

## Appendix B

### Barbour Classification

The 7 heptatonic scale classes listed in Table B.2 are those given by Barbour (Barbour, 1949, p. 587). They correspond to all 66 families across all 13 parts of the catalogue, as shown in Table B.1 and Figure B.1, distinguished by Barbour's scale-step vector (see Additional Notes below).

Table B.1: The distribution of families by width, distinguished by Barbour's scale-step vector.

Vector	Width													Total
	7	9	10	11	12	14	16	17	19	21	24	26	31	
25	1	1		1										3
331			2	2	6	6	4							20
412				1	2	2	4		3	3				15
4201								6	6	3				15
5011								2		2		2		6
51001											4	2		6
600001													1	1
Total	1	1	2	4	8	8	8	8	9	8	4	4	1	66

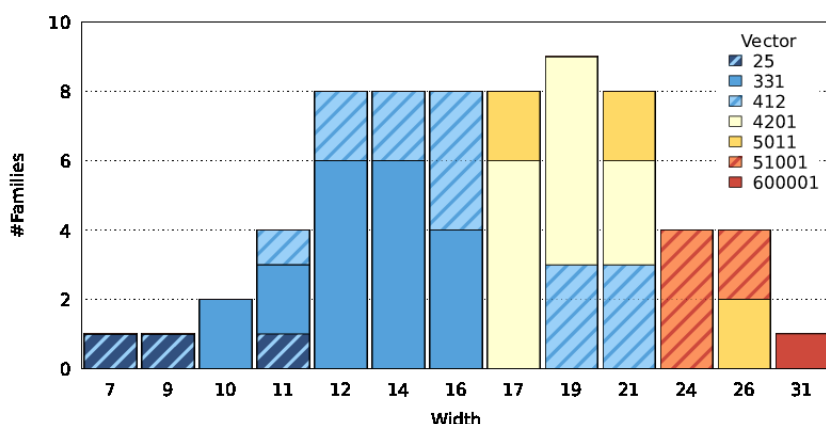


Figure B.1: The distribution of families by width, distinguished by Barbour's scale-step vector.

## Appendix C

### Scale-Step Lookup

Each of the 462 scales in this catalogue has a unique scale-step sequence (see Scale Properties, p. 14). All 462 scale-step sequences are listed below, each with a reference to the corresponding scale in the catalogue.

When looking up a scale-step sequence, ensure that it conforms to the required form – that is, it consists of exactly 7 steps, each between 1 and 6 semitones in size and always totalling 12 semitones. The scale-step sequences are listed in ascending numerical order. Consequently, scales near the start or end of this list occur towards the end of the catalogue, while scales near the middle of this list occur towards the start of the catalogue.

1111116 (§66.1)	1112214 (§35.7)	1115112 (§59.1)
1111125 (§65.1)	1112223 (§28.3)	1115121 (§61.3)
1111134 (§62.1)	1112232 (§32.2)	1115211 (§64.4)
1111143 (§63.2)	1112241 (§54.3)	1116111 (§66.4)
1111152 (§64.2)	1112313 (§29.4)	1121115 (§59.5)
1111161 (§66.2)	1112322 (§31.3)	1121124 (§47.7)
1111215 (§60.2)	1112331 (§55.4)	1121133 (§41.5)
1111224 (§54.2)	1112412 (§49.3)	1121142 (§46.3)
1111233 (§55.3)	1112421 (§57.3)	1121151 (§58.5)
1111242 (§57.2)	1112511 (§65.3)	1121214 (§37.5)
1111251 (§65.2)	1113114 (§38.1)	1121223 (§18.5)
1111314 (§51.7)	1113123 (§25.1)	1121232 (§24.5)
1111323 (§50.2)	1113132 (§30.1)	1121241 (§45.5)
1111332 (§56.1)	1113141 (§51.1)	1121313 (§19.4)
1111341 (§62.2)	1113213 (§26.4)	1121322 (§21.3)
1111413 (§52.4)	1113222 (§27.2)	1121331 (§42.4)
1111422 (§53.2)	1113231 (§50.3)	1121412 (§40.3)
1111431 (§63.3)	1113312 (§43.2)	1121421 (§48.3)
1111512 (§61.2)	1113321 (§56.2)	1121511 (§60.4)
1111521 (§64.3)	1113411 (§62.3)	1122114 (§33.1)
1111611 (§66.3)	1114113 (§39.4)	1122123 (§10.1)
1112115 (§58.4)	1114122 (§34.5)	1122132 (§14.1)
1112124 (§45.4)	1114131 (§52.5)	1122141 (§35.1)
1112133 (§42.3)	1114212 (§44.1)	1122213 (§6.4)
1112142 (§48.2)	1114221 (§53.3)	1122222 (§7.3)
1112151 (§60.3)	1114311 (§63.4)	1122231 (§28.4)

## Appendix D

# Ethnomusicology

### Overview

The five sections in this appendix correspond to five ethnomusicological scale classifications – the South-Indian *Mela*, North-Indian *Thāt*, Arab *Maqām*, Turkish *Makam* and Greek *Dromos* systems. Each section provides a table and figure showing how the corresponding scales and families are distributed across this catalogue, and a main lookup table that lists references to individual scales. Accordingly, in the main body of the catalogue, any scale with ethnomusicological associations lists them with references into the sections of this appendix.

Of the 462 scales in the catalogue, 81 scales have ethnomusicological associations. Of these, 36 scales have multiple associations. This is not to suggest that all these associations can simply be reduced to the same scale structure; rather, each association uses the structure in a different way. Identifying the underlying commonality of scale structure eliminates it as a distinguishing feature, and allows other distinctive ‘modal’ characteristics to be given greater emphasis, highlighting the crucial distinction between ‘scale’ and ‘mode’ (Powers, 2001, pp. 776–777).

In this context, the South-Indian *mela* and North-Indian *thāt* are scale classifications (with modal aspects captured in the higher-level concept of *rāga*), and the Greek *dromos* is also essentially a scale classification. Conversely, the Arab *maqām* and Turkish *makam* are modal classifications.

The distinction between scale and mode is apparent in the choice of tonic<sup>1</sup>. Each *mela* and *thāt* has an implied tonic of C and each *dromos* has an implied tonic of D, emphasising scale structure. In contrast, each *maqām* and *makam* has its own explicit tonic note, emphasising position.

This emphasis on scale position reflects that “modulation forms a prominent feature of Arab music performance... in contrast to the situation in North and South Indian classical music, where modulation is not prominent” (Marcus, 2002, pp. 41–42; see also Marcus, 1992). Modulation also features extensively in the Turkish *makam* tradition (Signell, 1986, p. 66).

Furthermore, the 72 *mela*, 35 *thāt* and 19 *dromoi* each correspond to a distinct scale structure within their respective systems, while the 29 *maqāmāt* and the 30 *makamlar* correspond to only 19 and 13 distinct scale

---

<sup>1</sup>The term ‘tonic’ is used here in the broad sense of ‘home note’ with no implications of Western ‘tonality’.

## D.1 South-Indian *Mela*

The 72 South-Indian *mela* listed in Table D.2 are those given by (Morris, 2004, pp. 75–76). They correspond to 72 scales from 36 families across the first 10 parts of the catalogue, as shown in Table D.1 and Figure D.1.

Table D.1: The distribution of the 36 families and 72 scales corresponding to the 72 South-Indian *mela*.

Width	7	9	10	11	12	14	16	17	19	21	24	26	31	Total
Families	1	1	2	4	8	5	6	5	2	2				36
Scales	6	4	7	12	15	9	10	5	2	2				72

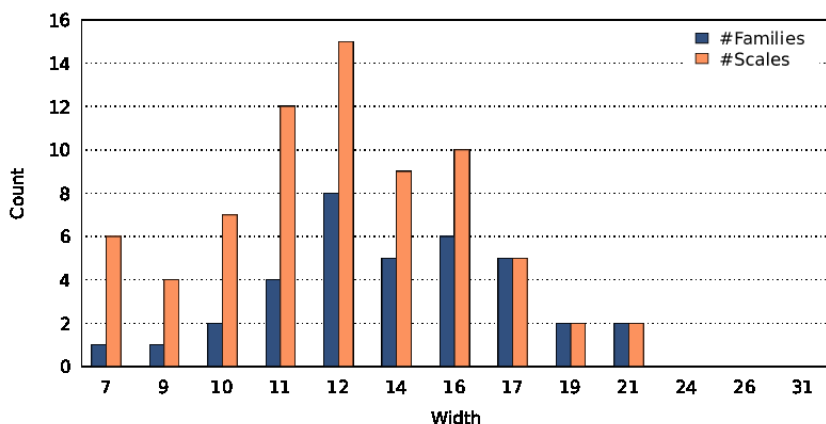


Figure D.1: The distribution of the 36 families and 72 scales corresponding to the 72 South-Indian *mela*.

Table D.2: The 72 South-Indian *mela* and their corresponding scales.

#	Name	Scale
1	Kanakangi	$\sharp 15.5$ Phrygian $\flat 7\flat 3$
2	Ratnangi	$\sharp 9.5$ Phrygian $\flat 3$
3	Ganamurti	$\sharp 26.5$ Phrygian $\sharp 7\flat 3$
4	Vanaspati	$\sharp 17.1$ Dorian $\flat 2\flat 3$ /Phrygian $\sharp 6\flat 3$
5	Manavati	$\sharp 27.3$ Ionian $\flat 2\flat 3$
6	Tanarupi	$\sharp 50.4$ Ionian $\flat 2\sharp 6\flat 3$ /Phrygian $\sharp 7\flat 3\sharp\sharp 6$
7	Senavati	$\sharp 5.5$ Phrygian $\flat 7$
8	Hanumatodi	$\sharp 1.5$ Phrygian
9	Dhenuka	$\sharp 6.5$ Phrygian $\sharp 7$

# Bibliography

- Abraham, Otto and Erich M. von Hornbostel (1994). "Suggested methods for the Transcription of Exotic Music". In: *Ethnomusicology* 38.3, pp. 425–456.
- Akkoç, Can, William A. Sethares, and M. Kemal Karaosmanoğlu (2015). "Experiments on the Relationship between Perde and Seyir in Turkish Makam Music". In: *Music Perception* 32.4, pp. 322–343.
- Antokoltez, Elliott (1993). "Transformations of a Special Non-Diatonic Mode in Twentieth-Century Music: Bartók, Stravinsky, Scriabin and Albrecht". In: *Music Analysis* 12.1, pp. 25–45.
- Barbour, J.M. (1929). "Synthetic Musical Scales". In: *American Mathematical Monthly* 36.3, pp. 155–160.
- (1949). "Musical Scales and Their Classification". In: *Journal of the Acoustical Society of America* 21.6, pp. 586–589.
- Barsky, Vladimir (1996). *Chromaticism*. Amsterdam: Harwood Academic.
- Bent, Ian (2001). "Notation". In: *The New Grove Dictionary of Music and Musicians*. Ed. by S. Sadie and J. Tyrrell. Vol. 18. London: Macmillan, pp. 73–189.
- Bent, Margaret and Alexander Silbiger (2001). "Musica ficta". In: *The New Grove Dictionary of Music and Musicians*. Ed. by S. Sadie and J. Tyrrell. Vol. 17. London: Macmillan, pp. 441–453.
- Bernard, Jonathan (1997). "Chord, Collection and Set in Twentieth-Century Theory". In: *Music Theory in Concept and Practice*. Ed. by James M. Baker, David W. Beach, and Jonathan W. Bernard. Rochester: University of Rochester Press, pp. 11–51.
- Busoni, Ferruccio (1907). *Entwurf einer neuen Ästhetik der Tonkunst*. Eng. trans. by Baker, Theodore as *Sketch of a New Esthetic of Music*, G. Schirmer, New York, 1911. Leipzig: Insel-Verlag.
- Byros, Vasili (2008). "Competing 'Windows of Order': The Dialectics of System-Construction and-Withdrawal in Berg's Sonata for Piano, Op. 1". In: *Theory and Practice* 33, pp. 273–327.
- Carpenter, Patricia (1997). "Tonality: A Conflict of Forces". In: *Music Theory in Concept and Practice*. Ed. by James M. Baker, David W. Beach,

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# About the book

The scale is one of the basic elements of music. From the very first "A-B-C" or "do-re-mi", the concept of the scale shapes how we fundamentally think about musical notes and how we name and organise them. Even music that radically departs from these foundations remains framed by patterns such as the black-and-white keys of the piano or the lines and spaces of musical stave notation.

Individual musical scales can also be considered in the wider context of theoretical questions such as:

- how many possible scales are there?
- how do these scales relate to each other?
- how far does each scale stretch conventional music notation?

This catalogue presents all of the 462 distinct heptatonic octave scales that are representable in conventional Western music notation. These scales have been carefully organised into a structured and fully cross-referenced framework, making it easy to navigate between related scales while always retaining a sense of where each scale fits within the bigger picture.

The catalogue's organisation maps out how the scales progressively stretch conventional music notation beyond its diatonic foundations. This provides a broader heptatonic perspective on chromaticism and its role in the transitional music between tonality and atonality.

Appendices provide links to the pitch-class sets of atonal theory, as well as ethnomusicological frameworks such as the Indian 'mela' and 'thāt' scale classifications, and the modes of the Arab 'maqām', Turkish 'makām' and Greek 'dromos' systems.

# About the author



Since graduating from the Department of Computing at Imperial College London in the early 1990s, Nicholas has worked in the IT industry as a CAD/CAM software developer, UNIX Systems Administrator, Enterprise Architect and Text and Data Mining Analyst. He is now an Independent Researcher and Software Developer specialising in Computational Musicology, with a passion for using technology to make music theory accessible and practical for all.



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